PROMOTING MATHEMATICAL THINKING IN THE MALAYSIAN CLASSROOM: ISSUES AND CHALLENGES

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One main aim of the Malaysian mathematics curriculum was "to develop individual who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decisions" (p.2, Ministry of Education Malaysia, 2003). This indicates the significant emphasis of mathematical thinking in the intended curriculum. However, to what degree is mathematical thinking being promoted in the real mathematics classroom? This paper will begin with a discussion of what is mathematical thinking and how mathematical thinking is defined in the Malaysian context. Based on related literature reviews, this paper will then discuss the extent in which mathematical thinking has been implemented in the mathematics classroom and highlight some issues and challenges faced by Malaysian mathematics teachers in their efforts to promote mathematical thinking. The paper concludes with some recommendations for promoting mathematical thinking in the classroom teaching such as planning effective lessons and activities through Lesson Study collaboration.

INTRODUCTION

What is mathematical thinking? Is mathematical thinking similar to 'think mathematically'? These are questions that strike our mind while we were reading and searching for literature related to mathematical thinking. Surprisingly, there is yet to find well defined meaning or explanation of mathematical thinking. According to Lutfiyya (1998), "mathematical thinking involves using mathematically rich thinking skills to understand ideas, discover relationships among the ideas, draw or support conditions about the ideas and their relationships and solve problem involving the ideas." (p. 55). Ideas here may refer to mathematical concepts or knowledge. Whereas Schoenfeld (1992) proposed that there are five important aspects of cognition involve in the inquiries of mathematical thinking and problem solving, namely (a) the knowledge base; (b) problem solving strategies; (c) monitoring and control; (d) beliefs and affects; and (e) practices (p.348). As for OEDC (2000), mathematical thinking is described as a process which involves distinguishing between different kinds of statements, such as definitions, theorem, conjecture, hypothesis, examples, condition assertions; posing of higher order problem; and knowing that the answers sound logic to the problem. Alternatively, Suzuki (1998) defined mathematical thinking as global concepts that include all the mathematical activities and traditional ways of solving routine mathematical problems.

Although all the above descriptions were not totally similar, they seem to highlight three major components of mathematical thinking: a) mathematical

content/knowledge; b) mental operations; and c) predisposition. Mathematical content/knowledge refers to the specific mathematics subject matter, mathematical concepts and ideas that one has acquired or learnt, while mental operations can be illustrated as cognitive activities that the mind needs to perform when thinking (Beyer, 1988). As for predisposition, it refers to the tendency or predilection to think in certain ways under certain circumstances (Siegel, 1999). Examples of predisposition include reasonableness, thinking alertness and open-mindedness, as well as beliefs and affects.

WORKING DEFINITION OF MATHEMATICAL THINKING

In view of the above discussion, we would propose that a working definition of mathematical thinking should include the following characteristics:

- it involves the manipulation of mental skills and strategies
- it is highly influenced by the tendencies, beliefs or attitudes of a thinker
- it shows the awareness and control of one's thinking such as metacognition
- it is a knowledge–dependent activities

Base on these characteristics, we would like to define mathematical thinking as a mental operation supported by mathematical knowledge and certain kind of predisposition, toward the attainment of solution to problem. The interrelationships among these variables which constitute mathematical thinking are displayed in Figure 1.

As shown in Figure 1, each dimension of mathematical thinking is interrelated and complements one another. For this reason, any effective mathematical thinking act will involve the orchestration of elements in these three components. Acquisition of content knowledge is the basis to engage in mathematical thinking. Understanding of subject matter will support and guide one to choose the appropriate cognitive skills and strategies according to the problem situation. However, the acquisition of knowledge requires one to explore, inquire, seek clarity, take intellectual risks, and think critically and imaginatively (Tishman, Jay & Perkins, 1993**)**. Hence, the right attitudes or dispositions toward attainment of content knowledge are very important and serve as the ground force to execute cognitive skills and strategies in mathematics problem–solving. In sum, to become a successful and effective mathematical thinker, one needs to possess and internalize all these three components: content knowledge, cognitive skills cum strategies and thinking dispositions.

Now, let us turn to see what was the definition or meaning of mathematical thinking as defined in the Malaysian school mathematics curriculum.

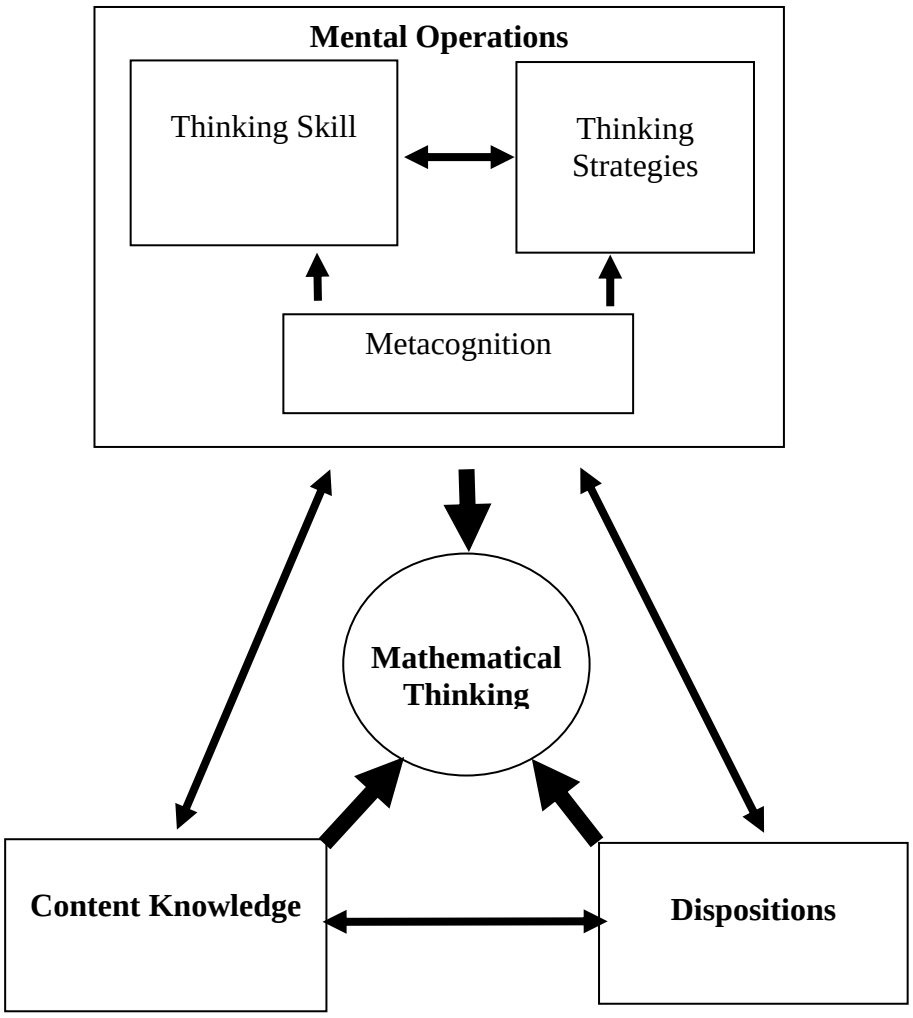

Figure 1: conceptual framework of mathematical thinking

MATHEMATICAL THINKING AS DEFINED IN MALAYSIAN MATHEMATICS CURRICULUM

A careful examination of the Malaysian school mathematics curriculum, both primary and secondary levels gave us a surprise. The word "mathematical thinking" was not used or stated explicitly in the Malaysian mathematics curriculum at all levels. Does this mean mathematical thinking is not an important component of the curriculum? However, we found a related statement in the secondary curriculum document. The word "to think mathematically" was used in the write-up of the main aim of secondary school mathematics curriculum as shown here:

> The Mathematics curriculum for secondary school aims to develop individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decision. (Ministry of Education Malaysia, 2003b, p.2)

The above statement denotes that mathematical thinking should be promoted in the Malaysian mathematics classroom if we are to produce future students who can think mathematically. This is especially pertinent to prepare future generations that is able

to face challenges in everyday life that arise due to the advancement of science and technology.

Table 1: Comparison of Mathematics Objectives between Primary School Curriculum and Secondary School Curriculum

Primary school mathematics*	Secondary school mathematics**
Objective 4: master basic mathematical skills, namely: making estimates and approximates; measuring; handling data; representing information in the form of graphs and charts	**Objective 3:** acquire basic mathematical skills such as: making estimation and rounding; measuring and constructing; collecting and handling data; representing and interpreting data; recognizing and representing relationship mathematically; using algorithm and relationship; solving problem; and making decision.
Objective 5: use mathematical skills and knowledge to solve problems in everyday life effectively and responsibly.	**Objective 5:** apply knowledge and the skills of mathematics in solving problems and making decisions **Objective 8:** cultivate mathematical knowledge and skills effectively and responsibly
Objective 6: use the language of mathematics correctly	**Objective 4:** communicate mathematically
Objective 8: apply the knowledge of mathematics systematically, heuristically, accurately and carefully	**Objective 6:** relate mathematics with other areas of knowledge
Objective 10: appreciate the importance and beauty of mathematics	**Objective 9:** Inculcate positive attitudes towards mathematics **Objective 10:** appreciate the importance and beauty of mathematics

*Ministry of Education Malaysia [MOE] (2003a).
**Ministry of Education Malaysia [MOE] (2003b).

Nonetheless, a closer analysis of the intended objectives shows that there are elements of mathematical thinking incorporated in both the primary and the

secondary school mathematics curriculum documents. Table 1 displays the related objectives.

Table 1 seems to show that all the three components of mathematical thinking are implicitly incorporated in both levels of Malaysian school mathematics curricula. For the primary mathematics curriculum, there is a higher emphasis on basic mathematical skills as compared to the problem solving skills and appreciation of mathematical values. In comparison, the emphasis is more on complex mathematical skills such as problem solving, decisions making, communication and extension of mathematical abstraction as well as positive attitudes toward mathematics rather then the basic mathematical skills for the secondary mathematics curriculum.

PROMOTING MATHEMATICAL THINKING IN THE CLASSROOM TEACHING

The above document analysis of curriculum indicates that promoting mathematical thinking among Malaysian students is an intended goal even though it was not explicitly spelled out in the syllabus. However, to what extent has mathematical thinking been successfully implemented in the Malaysian classrooms?

A search of local literatures shows that research concerning mathematical thinking is still very limited. There were two related articles (see Yudariah and Tall, 1995; Roselainy, Yudariah and Mason, 2002) that have the words “mathematical thinking” in their titles. In the first study, Yudariah and Tall (1995) compared the professors’ perceptions of students’ mathematical thinking between what they expect and what they prefer. However, the mathematical thinking was inferred from the students’ attitudes towards mathematics and problem solving. There was no clear definition of mathematical thinking.

While in the second study, Roselainy, Yudariah and Mason (2002) reported a study that aimed to invoke 49 engineering undergraduates’ mathematical thinking through the teaching of differentiation. They attempted to promote students’ mathematical thinking by engaging them in ‘various kinds of mental activities that signify mathematical thinking (specializing, exemplifying, generalizing, conjecturing and convincing)…’(p.288). For example, in the teaching of the chain rule, students were given various examples of class of functions that succumb to the Chain Rule. The students’ attention were then ‘directed towards identifying “what stays the same”, “what is different” and “what can be changed” as well as encouraged to “say what they see”’ (p.288). They believed that by ‘explicitly connect the mathematical structures (definition, properties, facts, example, technique) to the process of mathematical thinking (exemplifying, specializing, generalizing) might invoke students to think mathematically. However, there was no report on whether the students’ mathematical thinking was enhanced after the study.

In brief, there is a need to have much more empirical study that focus on promoting mathematical thinking in the Malaysian classroom. This is especially so if teaching

of mathematical thinking is one of the intended goals in the Malaysian school mathematics curriculum.

ISSUES AND CHALLENGES IN PROMOTING MATHEMATICAL THINKING

From our observation and experience as former mathematics teachers in schools, some mathematics teachers do attempt to integrate mathematical thinking in their lessons. One mathematics master teacher that I interviewed recently, elaborated that he used to promote mathematical thinking "indirectly and unintentionally through questioning, discussion, problem solving and projects". Another mathematics teacher explained that he tried to promote mathematical thinking when teaching students to solve word problems. He would ask his students to think what the question want; what information given; how are they going to solve it. He also stressed the importance of giving students sufficient time to think and to guide them to solve on the board. Nevertheless, informal discussion with other mathematics teachers also indicates that many mathematics teachers agreed to the importance of mathematical thinking and would like to promote mathematical thinking in their classrooms. But they are usually constrained by several issues and challenges as discussed below:

Issue 1: no clear understanding of mathematical thinking

As mentioned earlier, there is no explicit or clear cut definition of mathematical thinking in the curriculum. One mathematics teacher opinioned that mathematical thinking was mentioned more during the orientation course given by the Curriculum Development Centre. Mathematical thinking was taught as related to higher order thinking, critical and analytical thinking as well as problem solving. Hence, there is generally a lack of clear understanding about what is mathematical thinking among Malaysian mathematics teachers**.** Many teachers perceived mathematical thinking to problem solving or higher level of questioning. Others referred mathematical thinking to critical and creative thinking skills.

For example, 'logical reasoning' was one of the mathematics topics in the upper secondary mathematics curriculum (MOE, 1998). Initially most teachers tended to teach the topic using daily life examples that emphasize on language structures such as "all animals are living things" and "some animals do not eat meat". Very often, the examples given were not directly related to mathematical ideas. Hence, in the later curriculum revision (MOE, 2000), it was changed to name as "mathematical reasoning". Teachers are expected to use examples that involved mathematical sentences, statements and symbols. For example, 'all trapeziums have two parallel sides.'; 'some even numbers are divisible by 4'. This aspect is also stressed in the mathematics textbooks. Yet, many teachers still tend to teach it following a certain rigid procedure, instead of encouraging students to estimate, predict and making intelligent guessing in the process of seeking solutions.

Issue 2: Examination oriented culture and 'finish syllabus syndrome'

The examination oriented culture is still prevalent in Malaysian schools, in spite of the government's effort to "humanize" the public assessment system recently. Examination results, especially the public examination result remain to be used as a yard stick or accountability of school performance. It is also common for school principals to use students' performance as appraisal to assess teachers' teaching performance. Under the pressure of achieving excellent examination results, it is not surprising to observe that most teachers tended to teach to test. They were more anxious to finish the syllabus so as to answer to the expectation of the school principal and parents, regardless of students' understanding and learning. This kind of "finish the syllabus syndrome" often render teachers no choice but to use procedural teaching that is a fast and direct way of information/knowledge transfer. Many teachers stress on "drill and practice" so that students are familiar with the style of examination questions. Students are taught to master the answering techniques, instead of executing mathematics thinking skills and strategies to solve the problems.

Issue 3: Lack of appropriate assessment instrument

In relation to the examination oriented culture, what is not assessed in the examination will not be taught in class. Analyses of the past year examination papers shows that there were very few questions that assess mathematical thinking. Even items that were categorized as problem solving were set in such a common format that they can be easily solved using a predicted model or procedure. Moreover, it is also common to find school based test papers adopting or adapting those of the commercial publishers. Yet the latter are usually modeling the past year examination items. Hence, this lack of appropriate instrument or examination items that assess mathematical thinking might be another reason of not promoting mathematical thinking in the classroom by many mathematics teachers.

Challenge 1: lack of resources and know-how in promoting mathematical thinking

Beside the lack of clear understanding about mathematical thinking, teachers generally do not receive enough support from their school, especially in terms of teaching and learning materials, references and professional development training. Furthermore, most teachers experienced their school mathematics learning through procedural approach. Many of them tended to teach as they were taught. Hence, many teachers still lack the know-how and resources to incorporate mathematical thinking activity in their mathematics lessons. They need extra time and effort in preparation, while time is the biggest constraint in view of the examination oriented culture and heavy workload of teachers. Consequently, this discourages many teachers from integrating mathematical thinking activity in their lessons.

Challenge 2: The role of technology in mathematical thinking

In line with the latest policy which changes the medium of instruction from Malay language to English language in the teaching of mathematics and science from the year 2003, the Malaysian Ministry of Education has supplied both hardware (e.g. notebook computer, graphic calculators) and software (such as Geometry Sketchpad) as well as teaching courseware and learning packages to the schools. The intention was to assist teachers to upgrade their pedagogical approach and to promote understanding of mathematical concepts. However, due to the low English language proficiency and insufficient competency in technology, some teachers were observed to merely exhibit the teaching courseware without much explanation or interaction with the students. There were teachers who just let their students "watch" the teaching program in the absence of the teachers. The use of technology and the change in the medium of instruction, thus, pose a great challenge to both mathematics teachers and the Malaysian government. How to ensure that all teachers are competent in both language and technology so that they can make full use of the interactive exploratory environments provided by technology? How to equip all teachers with the knowledge of choosing the relevant software and appropriate technological tools that will encourage students to think mathematically? These are questions that need immediate attention.

SUGGESTIONS FOR PROMOTING MATHEMATICAL THINKING

In spite of the above issues and challenges, we believe all mathematics teachers and educators will agree to the importance of promoting mathematics thinking in mathematics lessons. Therefore, we would like to offer some suggestions here.

(a) Equip and enhance mathematics teachers' understanding of mathematical thinking

A more explicit and comprehensive explanation of mathematical thinking will have to be stated in the school mathematics curriculum documents so that teachers can referred to these documents. Pre-service and in-service mathematics teachers need to be made aware of the importance of mathematical thinking. They also need to be equipped with learning and to experience for themselves in mathematical thinking activities. These can be achieved by exposing mathematics teachers to various teaching strategies and activities that promote mathematical thinking. These ideas and activities can be imparted from time to time through workshops, seminars or conferences.

(b) Preparing mathematical thinking lessons through Lesson Study collaboration

As we all realized mathematics lessons that promote mathematical thinking usually take enormous time and effort to prepare. Through Lesson Study group, each group member can collaborate to plan, discuss and prepare the lesson. One of the group members can then teach the lesson while others observed. After the teaching session,

every member can gather together to reflect and revise the lesson. This kind of collaborative effort will certainly reduce the workload and time taken in preparing the lesson. More importantly, teachers will gain deeper understanding and more effective strategies through peer support. Consequently, more mathematics teachers might be more confident and more encouraged to integrate mathematical thinking in their future lessons.

(c) Redesign assessment framework that focus on mathematics thinking

As we observed earlier most teachers teach according to the requirement of the public examination. As long as the assessment framework do not emphasis on mathematical thinking, it would be very unlikely to see mathematics teachers willing to spend time to integrate mathematical thinking in their lessons. Hence, to achieve the intended goal, it is time for the Malaysian Ministry of Education to redesign the assessment framework that focus on mathematics thinking. With the new framework, it is hoped that mathematics teachers will then restructure their teaching approach so that promotion of mathematical thinking will become an essential component in their mathematics classroom teaching.

CONCLUSION

In this paper, we attempt to search for a clear definition of mathematics thinking in the Malaysian school mathematics curriculum. However, we were rather disappointed to find mathematics thinking was not explicitly explained in the intended curriculum, though it was stated as an objective in the secondary school mathematics curriculum. There were still very limited studies with regard to mathematics thinking in the local literatures. Through informal interviews with several mathematics teachers, we discussed three issues and three challenges that Malaysian mathematics teachers faced in the effort to promote mathematics thinking. We then offer three suggestions that we hope might help to resolve some of the issues and consequently promote mathematics thinking in the mathematics classroom. In sum, it is pertinent to promote students' mathematics thinking in mathematics classroom. To achieve that, there is an urgent need to make significant changes in our mathematics teaching and assessment framework that incorporate attributes of mathematics thinking.

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